Problems of Complexity in Cybernetics and Creation of Universal System of Knowledge

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Проблеми Складності в Кібернетиці та Створення Універсальної Системи Знань

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***Abstract*—Problems of complexity in modern cybernetics and computer science are discussed. Basic ways of its resolution is analyzed. It shown that this problem has more global value. Polymetric analysis as variant resolution of problem simplicity – complexity is represented.**

***Анотація*—Обговорюється проблема складності в кібернетиці nf syajhvfnbws. Аналізуються основні шляхи її розв'язання. Показано, що ця проблема має більш глобальне значення. Наведений поліметричний аналіз як варіант розв'язання проблеми простоти – складності.**

***Keywords*—complexity; cybernetics; S. Beer; polymetric analysis; hybrid theory of systems.**

***Ключові слова*—складність; кібернетика; С. Бір; поліметричний аналіз; гібридна теорія систем.**

I. Introduction

Problem simplicity-complexity is central problem for each science. Roughly speaking it is the problem of optimal formalization of knowledge. This problem is beginning from famous Archimedes phrase: "Give me fulcrum and I'll reverse Universe" [1]. Therefore, this problem is basic for the creation new science. It was used by Aristotle, Descartes, Newton and other researches.

So problem of complexity is one of central problem in modern mathematics and cybernetics [1 – 8]. This problem is caused in synthetically sciences. Roughly speaking it has two aspects: system (problem of century in cybernetics according S. Beer [1, 3]) and computational (problem of computational complexity [1, 5]. Last problem is included in basic problems of modern mathematics (Smale problems) [1, 5].

As variant of resolution system aspect of problem complexity in cybernetics may be problem simplicity – complexity, which is included in Polymetric Analysis (PA) (universal system of analysis, synthesis and formalization of knowledge) as principle simplicity. Basic elements of this theory – functional numbers is generalizing of quadratic forms.

Hybrid theory of systems (HTS) as element of PA is created on the basis principles (criteria) of reciprocity and simplicity [1, 2, 9]. Only 10 minimal types of formalization system may be used. But number of real systems may be infinite. These systems are differed by step of its complexity. It is may be represented as answer on the one of basic question of modern theory of systems [1, 2, 9] about possible number of systems and its classification with point of simplicity – complexity [1, 2, 9].

Therefore HTS may be represented as variant of resolution the problem of century in cybernetics according S. Beer and may be used for the resolution problem of computational complexity (theory of informative calculations, TIC) [1, 2, 9].

Theory of informative calculations may be represented as variant of resolution of computational complexity [1, 2, 9].

II. Basic Results

We begin this chapter from phrase by S. Beer [3]: "Apparently, the complexity becomes the problem of the century, just as the ability to process natural materials has been a problem of life and death for our forefathers. Our tool must be computers, and their efficiency should be provided by science, able to handle large and complex systems of probabilistic nature. This science may be cybernetics - the science of management processes and communication. The basic thesis of cybernetics can be set forth as follows: there are natural laws behavior of the large manifold systems of any character submits that – biological, technical, special and economic." But nature of some problems may be improbable too. Therefore this problem must be expanded on problem of formalization of all science and knowledge. But modern science is the realization of the R. Bacon – Descartian concept "Science is so science, how many mathematics is in her" [1]. Development of modern science practically isn't possible without computers.

Computational complexity theory [1, 5] is a branch of the theory of computation in theoretical computer science that focuses on classifying computational problems according to their inherent difficulty, and relating those classes to each other. A computational problem is understood to be a task that is in principle amenable to being solved by a computer, which is equivalent to stating that the problem may be solved by mechanical application of mathematical steps, such as an algorithm. Basic problem of computing complexity is connected with polynomial calculations.

Computational complexity is a concept in computer science and theory of algorithms, depending on the function of indicating the volume of work that is performed by some algorithm, the size of the input data [1, 5]. The science, which studies the computational complexity, was called the theory of computational complexity. The volume of work is usually measured by abstract concepts of time and space, called computational resources. The time is determined by the number of elementary steps needed to solve the problem, while the space defined by memory capacity, or space on the storage medium.

A problem is regarded as inherently difficult if its solution requires significant resources, whatever the algorithm used. The theory formalizes this intuition, by introducing mathematical models of computation to study these problems and quantifying the amount of resources needed to solve them, such as time and storage. Other complexity measures are also used, such as the amount of communication (used in communication complexity), the number of gates in a circuit (used in circuit complexity) and the number of processors (used in parallel computing). One of the roles of computational complexity theory is to determine the practical limits on what computers can and cannot do [1, 5].

Closely related fields in theoretical computer science are analysis of algorithms and computability theory. A key distinction between analysis of algorithms and computational complexity theory is that the former is devoted to analyzing the amount of resources needed by a particular algorithm to solve a problem, whereas the latter asks a more general question about all possible algorithms that could be used to solve the same problem. More precisely, it tries to classify problems that can or cannot be solved with appropriately restricted resources. In turn, imposing restrictions on the available resources is what distinguishes computational complexity from computability theory: the latter theory asks what kind of problems can, in principle, be solved algorithmically [1].

The question why the concept of computational complexity is hard for the verifiable mathematics was discussed by J. Hromkovič [8]. Therefore we must expand this problem on all science with help system with variable hierarchy or variable measure. And therefore we go to polymetric analysis.

III. Polymetric Analysis and Problem of Complexity

Polymetric analysis (PA) was created as alternative optimal concept to logical, formal and constructive conceptions of modern mathematics and theory of information [1, 10]. This concept is based on the idea of triple minimum: mathematical, methodological and concrete scientific [1, 9].

However, one of the main tasks of polymetric analysis is the problem of simplicity-complexity that arises when creating or solving a particular problem or science. It must be open system [1, 2, 9].

The polymetric analysis may be represented as universal theory of synthesis in Descartian sense. For resolution of this problem we must select basic notions and concepts, which are corresponded to PA. The universal simple value is unit symbol, but this symbol must be connected with calculation. Therefore it must be number. For the compositions of these symbols (numbers) in one system we must use system control and operations (mathematical operations or transformations). After this procedure we received the proper measure, which is corresponding system of knowledge and science.

Therefore the basic axiomatic of the polymetric analysis is was selected in the next form [1, 2, 9].

Definition 1. **Mathematical construction** is called set all possible elements, operations and transformations for resolution corresponding problem. The basic functional elements of this construction are called constructive elements.

Definition 2. The mathematical constructive elements $N_{x_{ij}}$ are called **the functional parameters**

$$N_{x_{ij}} = x_i \cdot \overline{x}_j , \qquad (1)$$

where x_i , $\overline{x_j}$ – the straight and opposite parameters, respectively; $\cdot$ – corresponding respective mathematical operation.

Definition 3. The mathematical constructive elements $N_{\varphi_{ij}}$ are called the **functional numbers**

$$N_{\varphi_{ij}} = \varphi_i \circ \overline{\varphi}_j . \qquad (2)$$

Where $\varphi_i\left(x_1,..x_n,\overline{x_1}..\overline{x_m},..N_{x_{ij}},\ldots\right)$, $\overline{\varphi_j}\left(x_1,..x_n,\overline{x_1}..\overline{x_m},..N_{x_{ij}},...\right)$ are the straight and opposite functions, respectively; $\circ$ – respective mathematical operation.

Remark 1. Functions φ_I , $\overline{\varphi_j}$ may be have different nature: mathematical, linguistic and other.

Another example may be the orthogonal eigenfunctions of the Hermitian operator.

The theory of generalizing mathematical transformations is created for works on functional numbers [1].

Definition 4. **Qualitative transformations** on functional numbers $N_{\varphi_{ij}}$ (straight A_i and opposite $\overline{A}_j$) are called the next transformations. The straight qualitative transformations are reduced the dimension $N_{\varphi_{ij}}$ on i units for straight parameters, and the opposite qualitative transformations are reduced the dimension $N_{\varphi_{ij}}$ on j units for opposite parameters.

Definition 5. **Quantitative (calculative) transformations** on functional numbers $N_{\varphi_{ij}}$ (straight O_k and opposite $\overline{O}_p$) are called the next transformations. The straight calculative trasformations are reduced $N_{\varphi_{ij}}$ or corresponding mathematical constructive element on k units its measure. The opposite quantitative transformations are increased $N_{\varphi_{ij}}$ or corresponding mathematical constructive element on l units its measure, i.e.

$$O_k\overline{O}_l N_{\varphi_{ij}} = N_{\varphi_{ij}} - k \oplus l. \quad (3)$$

Definition 8. **Left and right transformations** are called transformations which act on left or right part of functional number respectively.

Definition 9. The maximal possible number corresponding transformations is called **the rang of this transformation**

$$rang(A_i\overline{A}_j N_{\varphi_{ij}}) = \max(i, j); \quad (4)$$

$$rang(O_k\overline{O}_p N_{\varphi_{ij}}) = \max(k, p). \quad (5)$$

Remark 2. The indexes *i,j, k,p* are called the steps of the corresponding transformations.

For this case we have finite number of generalizing transformations.

Basic elements of PA were called the generalizing mathematical elements or its various presentations – informative knots [1, 2, 9]. Generalizing mathematical element ${}^{stqo}_{nmab}M_{ijkp}$ is the composition of functional numbers (generalizing quadratic forms, including complex numbers and functions) and generalizing mathematical transformations, which are acted on these functional numbers in whole or its elements [1]. Roughly speaking these elements are elements of functional matrixes.

This element ${}^{stqo}_{nmab}M_{ijkp}$ may be represented in next form

$${}^{stqo}_{nmab}M_{ijkp} = A_i\overline{A}_j O_k\overline{O}_p A^r_s\overline{A}^r_t O^r_q\overline{O}^r_o A^l_n\overline{A}^l_m O^l_a\overline{O}^l_b N_{\varphi_{ij}}. \quad (6)$$

Where $N_{\varphi_{ij}}$ – functional number; $O_k, O^r_q, O^l_a, \overline{O}_p, \overline{O}^r_o, \overline{O}^l_b$; $A_i, A^r_s, A^l_n, \overline{A}_j, \overline{A}^r_t, \overline{A}^l_m$ are quantitative and qualitative transformations, straight and inverse (with tilde), (r) – right and (l) – left..

We have only 15 minimal types of mathematical transformations, only six are mathematicval in classic sense.

Polyfunctional matrix, which is constructed on elements (6) is called informative lattice. For this case generalizing mathematical element was called knot of informative lattice [1, 2, 9]. Informative lattice is basic set of theory of informative calculations. This theory was constructed analogously to the analytical mechanics [1].

Basic elements of this theory are [1, 2, 9]:

1) Informative computability C is number of possible mathematical operations, which are required for the resolution of proper problem.

2) Technical informative computability $C_t = C\sum t_i$, where t_i – realization time of proper computation.

3) Generalizing technical informative computability $C_{t0} = k_{ac}C_t$, where k_{ac} – a coefficient of algorithmic complexity [1].

Basic principle of this theory is **the principle of optimal informative calculations** [1, 2, 9]: any algebraic, including constructive, informative problem has optimal resolution for minimum informative computability C, technical informative computability C_t or generalizing technical informative computability C_{to}.

For classification the computations on informative lattices hybrid theory of systems was created [1]. This theory allow to analyze proper system with point of view of its complexity,

The basic principles of hybrid theory of systems are next: 1) **the criterion of reciprocity;** 2) **the criterion of simplicity**.

The criterion of reciprocity is the principle of the creation the corresponding mathematical constructive system (informative lattice). The criterion of simplicity is the principle the optimization of this creation.

The basic axiomatic of hybrid theory of systems is represented below.

Definition 8. The set of functional numbers and generalizing transformations together with principles reciprocity and simplicity (informative lattice) is called **the hybrid theory of systems** (in more narrow sense the criterion of the reciprocity and principle of optimal informative calculations).

Criterion of the reciprocity for corresponding systems is signed the conservation in these systems the next categories:

1) the completeness;

2) the equilibrium;

3) the equality of the number epistemological equivalent known and unknown knotions.

Criterion of the simplicity for corresponding systems is signed the conservation in these systems the next categories:

1) the completeness;

2) the equilibrium;

3) the principle of the optimal calculative transformations.

For more full formalization the all famous regions of knowledge and science the **parameter of connectedness σ_t** was introduced. This parameter is meant the number of different bounds the one element of mathematical construction with other elements of this construction. For example, in classic mathematics $\sigma_t = 1$, in linguistics and semiotics $\sigma_t > 1$. The parameter of connectedness is the basic element for synthesis in one system of formalization the all famous regions of knowledge and science. It is one of the basic elements for creation the theory of functional logical automata too.

Thus we can receive next 10 types of hybrid systems [1, 2, 9]:

1. The system with conservation all positions the criteria of reciprocity and simplicity for all elements of mathematical construction ($N_{\varphi_{ij}}$ and transformations) is called the *simple system.*

2. The system with conservation the criterion of simplicity only for $N_{\varphi_{ij}}$ is called the *parametric simple system.*

Remark 4. Further in this classification reminder of criteria of reciprocity and simplicity is absented. It mean that these criteria for next types of hybrid systems are true.

3. The system with conservation the criterion of simplicity only for general mathematical transformations is called *functional simple system.*

4. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t = 1$ is called the *semisimple system.*

5. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ and with $\sigma_t = 1$ is called the *parametric semisimple system.*

6. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t = 1$ is called the *functional semisimple system.*

7. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t \neq 1$ is called *complicated system.*

8. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ is called *parametric complicated system.*

9. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t \neq 1$ is called *functional complicated system*.

10. The system with nonconservation the criteria of reciprocity and simplicity and with $\sigma_t \neq 1$ is called *absolute complicated system*.

With taking into account 15 basic types of generalized mathematical transformations we have 150 types of hybrid systems; practically 150 types of the formalization and modeling of knowledge and science. Only first four types of hybrid systems may be considered as mathematical, last four types are not mathematically. Therefore HTS may be describing all possible system of knowledge. Problem of verbal and nonverbal systems of knowledge is controlled with help of types the mathematical transformations and parameter connectedness [1]. HTS allows classifying all possible knowledge by step simplicity – complexity. It may be used for the representation evolution of development of concrete science from complexity to simplicity. PA may be represented as natural concept of foundation of mathematics. It may be represented as formalization Pythagorean phrase "Numbers rules of the World", Plato concept of three types of numbers: arithmetical (pure mathematics); sensitive (applied mathematics) and ideal (numerology), and variant resolution S. Beer problem in more widely sense [1].

IV. Conclusions

The problem of simplicity – complexity in modern science is discussed. Short historical analysis of this problem is represented. Problems of complexity in modern cybernetics, computer science and mathematics are analyzed. Necessity of creation universal theory for resolution of problem simplicity – complexity in modern science is shown. Basic structure of PA is represented. Basic applications of PA for the resolution of some problems of modern cybernetics and science are analyzed. The questions about the presentation PA as natural concept of foundations mathematics and foundation of science in Descartian sense are analyzed.

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